Popular Computing

Volume 8 Number 12								
	December 198	0						
1234567890124567890012456789000000000000000000000000000000000000	7 70 777 700 777 7000 777777777 70 77 7000 77770 7000 7777 7000 7777 770 7007 770 770 770 770 770 770 770 770 770 777 770 777 777 770 777 770 777 770 777 770 777 770 777 770 777 770 777 770 7777 770 7777 770 7777 770 7777 770 7777 770 7777 770 7777 770 7777 770 7777 7770 7777 7770 7777 7770 7777 7770 7777 7770 7777	890123456789012345678901234 444444555555555666666666777777	770070 70707 7000 77777 7700 77777 7700 770770	The 7s Problem				

An Old Problem

Any integer, K, divides some number, N, that consists entirely of the digits O and 7. That fact was proved by David Ferguson in our issue number 37, April 1976.*

Clearly, for each K there are infinitely many N's. The table on the cover shows the smallest value of N for the first 74 values of K. The table was calculated by Mark Lewis Spitz, Reseda, California.

Mr. Spitz built up dividends as binary numbers, then changed each 1 to a 7, and tested the resulting number for divisibility by K. By incorporating several shortcuts (e.g., factors of 2 or 5 in K force a low-order zero in N), the table was produced quickly.

The solution is a brute force one, however, and would become impractical for many larger values of K (one notes that multiples of 9 appear to be particularly difficult).

So our Problem 280 is to devise a more efficient algorithm to extend the table.

*The cover of that issue contained the prediction that pocket computers would be available for \$250 by September of 1980. Fred Gluckson points out that we were off by one month and one dollar: the Radio Shack pocket machine sells for \$249.

Publisher: Audrey Gruenberger Editor: Fred Gruenberger Associate Editors: David Babcock

Irwin Greenwald Patrick Hall

Contributing Editors: Richard Andree

William C. McGee Thomas R. Parkin Edward Ryan

Art Director: John G. Scott Business Manager: Ben Moore POPULAR COMPUTING is published monthly at Box 272, Calabasas, California 91302. Subscription rate in the United States is \$20.50 per year, or \$17.50 if remittance accompanies the order. For Canada and Mexico, add \$1.50 per year. For all other countries, add \$3.50 per year. Back issues \$2.50 each. Copyright 1980 by POPULAR COMPUTING.

@ 2023 This work is licensed under CC BY-NC-SA 4.0

2 Popular Computing

For any positive integral value of R, what is the smallest power of 2 that contains R embedded contiguous zeros?

The status of the problem (which dates back to 1951) at that time was this:

R	Х
12345678	10 53 242 373 as 6081 1491 1492 6801
9	14007 unknown, but greater than 60000.

The entry for case 9 can now be filled in: 100823. In this number of 30351 digits, the nine contiguous zeros occupy the 21896 through 21904 positions. The computer run to gain these facts executed over 53 billion instructions.

The single- and double-digit distributions of the digits of the 100823rd power of 2 are given.

2936	2895	3136	3140	3063	2953	3131	3060	0 304	1 2	996		
0	1	2	3	4	5	6	7	8		9		
		01 2 2 3 3 3 3 4 0 5 6 0 7 0 8 6 0 9 10 2 11 12 13 14 15	285 17 281 18 294 19 299 20 21 266 22 281 25 281 25 279 27 288 28 297 29 281 31 274 32 318 33	3 292 292 267 277 2 311 3 341 3 310 3 327 3 321 3 301 3 302 298 2 309	34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50	316 319 313 312 334 309 321 300 313 303 282 299 324 291 309 281	52 534 556 578 590 666 666 666 666	285 312 305 315 260 316 308 276 295 313 307 337 337 337 337 337 337 337 337 33	68 70 71 77 77 77 77 77 81 82 83 84	310 318 293 296 305 3314 297 303 328 281 289 289 289 289 289 289 299	856 888 890 991 993 9956 999 999 999	308 312 313 303 289 314 274 308 293 283 293 283 309 313 267

+27	+2	-10	+2	+2	+1	Ŧ	+2	-11	6-	+3	-10	-5	-1	7
7	2	ů.	<u>د</u> -	ئ	-3	+111	βļ	-11	6-	Ŧ	42	6+	7+	-j
-10	4	-2	-5	+3	-5	-	+5	+5	+1	7	+3	\$	7+	e,
-10	1-	2	7-	+3	5	2-	+1	+13	47	9-	4	+	9	9
+10	1+	-10	£	+3	+5	-7	+1	-10	+5	-2	+3	7	+10	-5
+3	-5	5	ţ.	'n	-5	+10	-5	7	む	-5	-5	42	8+	-3
-3	-3	+10	+10	5	-10	1+	15	-10	5	+3	5	4	\$	r,
+2	٣-	+10	Ŧ	7	2+	+5	+1	+5	<u>ب</u>	+3	-5	+11	5	5-
+2	-5	+10	٣,	+2	+5	+11	+2	4	ň	+3	-11	7	-5	42
+2	+5	+10	ش	+1	+5	6-	45	9-	m,	+3	ŧ.	2-	-5	+10
+2	+2	-5-	m	7	7		-2	-5	9	£	-20	7	+4	+10
+3	+10	80	m-	-5	+2	-1	e,	-10	9+	+3	9+	-	4	+10
89	42	+3	+3	3	+5	-1	£-	æρ	9	48	5-	7	+10	+10
-10	9	+5	+3	2	-10	+3	-3	9-	77	-10	5	+10	+10	+3
+3	48	+5	+5	-2	42	-10	1+	7-	7-	7	5	+10	7	+3
+3	-2	£.	+3	7+	٠,	-10	\$	9	1-1	7	-10	+2	4-	4-
+2	e,	+5	+2	+2	-5	4	9	7	-10	7	-5	12	7-	7
7	7-	-5	+5	7+	-5	2	4	7	77+	48	+3	-11	+5	7
4	-3	-1	-1	7+	-5	-5	-2	-1	42	7	4	+5	-1	+3
7 +	+5	+5	+2	+1	-5	+10	+5	-1	+5	4	+3	+3	-1	7+



Traverse

The 10 x 20 grid on the facing page contains a collection of more-or-less random plus and minus numbers.

A path is to be found, starting at the upper left cell and arriving eventually at the lower left cell. This path is to proceed only orthogonally (that is, never diagonally); it must never cross itself; and is to have the property that the progressive totals of the numbers in the cells that it passes through are always composite numbers (that is, the sum must never be prime).

For example, if the path begins by going straight down, the progressive totals would be 4, 6, 8, 10, 11, and the 11 is not allowed, being prime.

Similarly, if the path begins by going straight across to the right, the progressive totals of the numbers that are crossed would be 4, 9, 10, 12, 13, and 13 is prime.

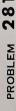
A total of zero should never be reached, and the number one is not prime for this exercise.

Note: the given pattern was constructed by first making a path that met the given conditions, and then filling in all the other cells with attractive looking numbers. Thus, there could be correct paths other than the one we had in mind.

So we know a priori that the problem has at least one solution. That leaves only two other problems:

- What is a possible algorithm for finding any solution?
- 2. Is it a suitable computer problem?

Popular Computing (5)



BOOK BEVOEW

The 6502 Microprocessor

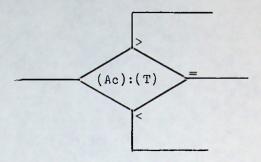
If you have an Apple, or a PET, or an AIM or KIM, then you have a 6502 microprocessor and you can write and execute programs in 6502 language. Usually, this is done via an assembler, since controlling the machine in absolute hex is possible, but it can get tedious and full of errors. You also need--most of the time--a good reference book, to explain the fine points of the processor, if only to refresh your memory. We will discuss here three such books:

First, there is the MOS Microcomputing Programming Manual (for which no author is credited), put out in 1976 by MOS Technology, Inc., the makers of the processor.

Then there is <u>Programming the 6502</u> by Rodnay Zaks, published by Sybex in 1978.

Finally, there is 6502 Assembly
Language Programming by Lance Leventhal,
published by Osborne/McGraw-Hill in 1979.

The edition of Zaks' book that is reviewed here came with an errata list of a hundred or so errors; this printed list should have one more entry: it is labelled "ERRATUM." In addition to the errors acknowledged in the list, the book is riddled with typos. Zaks' book is completely useless (worse, highly misleading) unless those errata are inserted into the text, but even then it is not too helpful. For example, the 6502 has the operation CMP, which is the COMPARE instruction. To be precise, it is a COMPARE ACCUMULATOR TO STORAGE type instruction, as in:



where (Ac) is the contents of the accumulator and (T) is the contents of the operand (say, a word of storage). The use of CMP normally calls for taking one of the following paths, for which the simplest operations are given here:

A = T	BEQ
$A \neq T$	BNE
A < T	BMI
A ≥ T	BPL
A > T	BNE followed by BPL
$A \leq T$	BEQ followed by BMI

It would be difficult to deduce all of this from the description given in Zaks' book.

Each of the three books lists all the op-codes in the 6502's repertoire; this is most likely the part of the book that the user will consult most frequently. It is illuminating to examine how each of the books treats an op-code. Picking one at random, let's see how ASL (Shift Left One Bit) is handled by each book.

The MOS book shows pictorially that a zero is generated into bit zero of the addressed word, and that bit 7 is moved to C (Carry). The N, Z, and C bits are enabled.

The ASL operation has 5 different op-codes, to permit zero page addressing, indexed addressing, or accumulator addressing. These are given, together with the assembly language form, and the number of execution cycles.

Zaks' book gives all the same information, but in a form that is much more difficult to read. For example, the op-code for ASL with absolute addressing is OE. Zaks shows that all the ASL commands have the form 000bbbl0, and that the form for absolute addressing has bbb = Oll, from which one may deduce the code OE. In addition, Zaks gives the data path for each op-code, but the diagrams require considerable study.

Leventhal's book includes, for each op-code, all the information one might need about it (some of it a bit difficult to dig out, especially when the list is not consulted frequently), but with one thing the other two books lack; namely, examples of the use of the command under various conditions.

The MOS book gives the bare facts with each op-code, and references back to the text material for more involved information.

Let's track another item through the three books. The 6502 has two index registers, called X and Y. They have similar uses and are controlled similarly, but

THEY ARE NOT INTERCHANGABLE.

The two most important differences are these:

- 1. Certain commands (such as the shift commands and the increment and decrement commands) are indexable only with X, not with Y.
- 2. The firmware of some microcomputers (specifically the Apple) uses the Y register and does not store and restore it. Thus, if the user has a loop controlled with the Y register and in that loop invokes any of the firmware, the program will do funny things.

In an 8-bit machine, good coding will make extensive use of the index registers. The novice, or the casual user, will expect much tutorial material on index registers, as well as all the facts about their use in practice. The MOS book does indeed have a long discussion about indexing, but it is only tantalizing about the X and Y registers of the 6502:

"Both index registers have the ability to be compared to memory (CPX, CPY) and to be incremented (INX, INY) and decremented (DEX, DEY). Because of OP CODE limitations, X and Y have slightly different uses. X is a little more flexible because it has Zero Page operations which Y does not have with exception of LDX and STX. Aside from which modes they modify, the registers are autonomous, independent and of equal value."

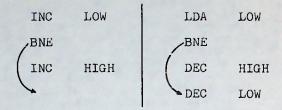
Zaks' book devotes a short paragraph to index registers; it says mainly that they are good things to have, and that they will be considered only in conjunction with indirect addressing.

Leventhal's book does not list index registers in its index and there is no specific section in the book on the subject of indexing. Leventhal does, however, tell more about the distinction between X and Y than either of the other two books:

"Only the X index register can be used for pre-indexed indirect addressing...Only the Y index register can be used with post-indexed indirect addressing...X and Y are not interchangeable since no instructions have both forms of simple indexing with both X and Y. In fact, the only instructions which allow zero-page indexing with Y are LDX and STX."

Zaks' book can be dismissed as fairly useless. For serious work with the 6502, the choice (if one must be made) is between the MOS book and Leventhal's. The former deals mainly with the architecture and pathology of the microprocessor; the latter, as its name implies, deals much more with assembly language programming. Leventhal's book refers incessantly to a prior book by Osborne (An Introduction to Microcomputing).

All three books are infuriating for avoiding the things the user wants most often. It is all very well and good to be given a tight loop to clear 100 words in page zero; what the user probably needs is a good routine to clear 10,000 words in general. Even elementary address modification:



is buried somewhere in all three books. Leventhal's book is rich with practical examples, but every one of them is written in page zero, which gets annoying. No one writes programs completely in page zero. To be sure, the user can figure out for himself how to rewrite the routines in upper memory, but the authors should do some of this.

Still, if a choice must be made, the recommendation is for Leventhal's book. The price is unknown, due to a phobia among publishers to reveal the retail price of a book to potential customers.



Problem numbers in recent issues were omitted as follows:

The Y-Sequence Issue 90, page 20.

Givoli's Problem Issue 91, page 14.

Newdice Toss
Issue 91, page 16.

Speaking of Records...

In issue 91 we reported on the calculation of factorial 11000, which was claimed as a record and was imprudently touted as "...in one sense, the...largest number ever calculated."

This brought a response from Harry Nelson, editor of the <u>Journal of Recreational Mathematics</u> and co-holder of the record for the world's largest known prime.

First, Mr. Nelson called attention to a report in 1965 in Mathematical Tables and other Aids to Computation, in a review by Dan Shanks, citing a solution to Archimedes' cattle problem consisting of a 206,545-digit integer. The solution is in the Unpublished Mathematical Tables file of MTAC.

Second, Mr. Nelson revealed that he had calculated every thousandth factorial from (1000!) to (1,000,000!) and sent along the first few digits of each of them, together with their size. Factorial ONE MILLION has 5,565,709 digits, and begins with 826393168...

His result for (11000!) agrees nicely with ours. Our calculation for the one minuscule factorial took 42 hours, while his calculation took around 10.5 hours for all thousand results.

At the time we first commented on Nelson's notable calculation of the 27th Mersenne prime (in our issue 81), we reprinted a letter that had appeared in the Los Angeles Times from a professor of astronomy, deploring the waste of (government) money and effort. Here is the reply that was sent to the professor from the public relations office at the Livermore laboratory:

I must agree with you that prime numbers don't have many practical applications. There are some, as Mr. Harry Nelson, one of the scientists here who discovered the 27th Mersenne Prime, pointed out recently. He told me that only three years ago there seemed to be no use for prime numbers as large as 100 digits. However, since then, scientists have found them useful in coding messages and the idea may one day be used in the country's national security program. Mostly, though, the hunt for larger prime numbers seems to be an exercise in basic scientific research.

You seemed concerned that time and money might have been wasted while our researchers looked for their prime number. I share your interest in frugality at government institutions. However, I do want to reassure you that in this case no time and money was wasted; the researcher's discovery was simply a delightful spinoff of work they would have been doing anyway.

The news story you read was probably abbreyiated from the attached press release. release explains that Mr. Nelson's job here is to test new computers for accuracy and reliability. When the Lab received a new Cray-1 computer last February, Mr. Nelson used David Slowinski's computer program to search for prime numbers as the new prime number was found. Mr. Nelson tells me that if he hadn't used Mr. Slowinski's program he would have used a similar diagnostic program with no practical value except to test the computer. As it turned out, the two researchers were able to make an interesting discovery while doing routine work, at no extra cost to the taxpayer. that all scientific research could be so economical.

> Sincerely, Linda Currey Public Information Office

Nelson himself offers these comments:

As to the present record lasting several decades, that depends upon the sequence of primes. If there is another Mersenne prime less than 100000, it will be found within two years, in my opinion. The program Slowinski and I wrote can be greatly improved (now that we are working at truly long precision) by fast-multiply methods. We know of one such method, easily applied to numbers in the 50000-bit range, which will result in an improvement of at least 3 and perhaps 5 over the version used to find M44497.

Incidentally, we stopped at 50023. Reiterating my main point above, it won't surprise me in the least if the primeness or compositeness of all Mersenne numbers up to 100000 is determined by the end of 1981. Media interest (including that of POPULAR COMPUTING) has an effect on such things.

and finally, to make the adventure complete, we have this reaction from the astronomy professor:

I read the news story in the Los Angeles <u>Times</u> when we were working on the budget for the School of Natural Sciences and literally trying to make ends meet. And if I recall correctly, the <u>Times</u> article led one to believe that two computer experts worked for three months to discover a 13,395-digit prime number. That prompted my letter to the Times.

With my past experiences with press releases, I should have been more careful; they are notorious for being inaccurate.

I was also aware of the possibility to use prime numbers in coding messages, but that morning it simply did not come to mind.

I hope my letter to the <u>Times</u> did not cause any inconvenience, and I certainly apologize for my hasty remarks. I am certain Mr. Nelson and Mr. Slowinski are performing good, basic research work; and I wish we had at California State University, Long Beach, a public information office staffed with someone with Ms Linda Currey's ability.

Answer to Last Month's Challenge Problem:

Most people are astonished to find that there is a tremendous difference which method of repayment is used and that the difference is relatively greater as the amount of the loan increases. One has only to calculate the interest on two loans-- of \$1000, and of \$1 000 000, say--for an odd number of years to see clearly what takes place.

Square Root Again

Webb Simmons, San Diego, California, suggests the use of the following algorithm for square root:

A = Argument $0 \le A \le 1$ S = Square rootD = Delta

D = Delt

Set S = 1/4

Set D = 1/4

LØØP: IF S > A GØTØ NEXT

 $A - S \longrightarrow A$

 $S + D + D \longrightarrow S$

*

NEXT $D/2 \longrightarrow D$ (Right shift, not divide)

 $S - D \longrightarrow S$

 $A + A \longrightarrow A$

IF D > O GØTØ LØØP **

The algorithm will usually terminate at the point marked (*), with the required root in S, controlled by a loop counter. The test at (**) is an alternative. If the procedure is controlled by a loop counter, then the test at (**) would be replaced by an unconditional branch to LØØP. The algorithm uses only add, subtract, shift, compare, and branch-all fast operations on any machine.

Mr. Simmons, who declares himself self-taught in computing, comments on his algorithm as follows:

On a computer, large or small, whose instruction set is limited to addition and subtraction for its mathematics, one must perform division by the well-known shift, test, and (possibly) subtract algorithm that develops the quotient one bit at a time. The square root algorithm above has its only utility in such a context; it develops the square root one bit at a time, and this only slightly slower than performing just one divide. Most assuredly my algorithm is not intended for use with a high level language except for demonstration purposes, and it would not be used in any language on computers like the Univac 1110, IBM 370, or CDC Cyber, etc., that has fast floating point hardware.

However, in a floating point computer environment, a square root routine often has a component module that takes the square root of a scaled value less than one. The argument to this module is derived from the mantissa part of the full argument that is supplied to the square root subroutine.

My first exposure to a computer square root method was on the Univac 1108 which had fast floating hardware. Its floating number is composed of a biased power of 2 as its characteristic (also called the power, or exponent) and a scaled fraction as its mantissa (also called factor or coefficient). The mantissa is always less than one but not less than one-half. For an odd characteristic, the square root algorithm increases the characteristic by one and divides the mantissa by 2; thus, the effective argument to the central square root algorithm is the adjusted mantissa: $1/4 \le \text{value} < 1$. The 1108 evaluates a polynomial to obtain a first guess with about 9 bits of accuracy; it then uses Heron's rule two times without further testing to obtain a full precision, normalized 27 bit mantissa (its word size is 36 bits with a 9 bit characteristic). The adjusted (possibly incremented) characteristic is then divided by 2 (except for the bias) and combined with the mantissa for the returned value.

My next exposure to the square root function on computers was on a Nova minicomputer which had no multiply or divide hardware. Its programmers decided to take a square root in much the same manner as on the 1108, but it was distressingly slow. I then devised an earlier version of the algorithm; I have placed this on a PDP-11 (without floating hardware) and on a 280 micro.

Along the way, I have had occasion to use Heron's rule, which can be derived as follows.

Assume that $x^2 \cong N$. If ϵ is the error, then

$$N \equiv (x + \epsilon)^2 = x^2 + 2\epsilon x + \epsilon^2 = x^2 + 2\epsilon x.$$

Solving for ϵ : $\epsilon = (N - x^2)/2x$

we should get a better approximation to $\sqrt{N} \cong x + \epsilon$

$$x_{n+1} = x_n + \frac{N - x_n^2}{2x_n} = x_n + (1/2) \left[\frac{N}{x_n} - x_n \right]$$

It is very interesting that you refer to my algorithm as "cleverly disguised" interval-halving. The action of my algorithm is best seen in binary, where the root is developed one bit at a time in much the same manner as a division. It is not interval-halving, but I suspect that the error decreases at about the same rate on the average because, at any time, the error is about 1/2 bit in the next bit position being developed. Lacking floating hardware, Newton's method cannot possibly be faster because you will need, usually, to or more divisions, and my method is almost as fast as one division.

Lately I have become interested in fast functional algorithms. I firmly believe that there are better ways than the classical methods introduced mainly by Hastings and his followers. For example, the CORDIC algorithm for trig functions (used on H-P and TI pocket calculators) is a real winner.

CORDIC means COrdinate Rotation DIgital Computer, but there is no need for the coordinates to rotate. The rotation had appeal in the context in which it was first employed (as a substitute for analog "resolvers" in aircraft navigational computers) but for ordinary math I have worked out a simpler approach in which the world sits still while a vector rotates. It is my guess that CORDIC has been neglected by the big computer world because (1) they had something already that worked, and (2) CORDIC is always presented, for some reason, in a strange, needless complicated manner.

Note: In our issue number 20, we presented a baker's dozen distinct algorithms for square root. Mr. Simmons seems to have given us a 14th method. We invite comments on it.

Problem Solution

Givoli's problem (number 278, in issue 91) called for finding the smallest progressive total of the primes (this is a sequence that begins 2, 5, 10, 17, 28, 41,...) that is a multiple of each successive integer.

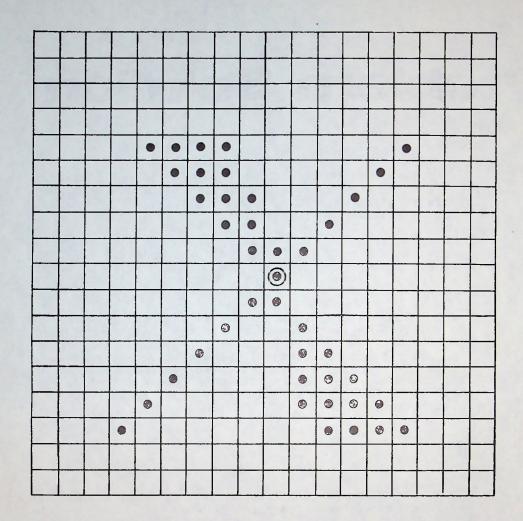
For example, for the integer 11, the 8th term of the sequence is a multiple (77), but for the integer 12 it is necessary to go to the 97th term to find the first exact multiple (22548). It is still an open question as to whether or not such a multiple can always be found.

It was stated in the article about the problem that no multiple had yet been found for the integers 303 or 326. The author of the problem, Nahman Givoli, of Tel Aviv, had noted that multiples of 6 were particularly difficult to calculate, and the two numbers mentioned were the last for which the results were unknown (that is, as far as the work had been carried), and they were not multiples of 6.

Ted Raab, Clifton Park, New York, reports that those entries to our table Y are now known:

d N S
303 1229 5,736,396
326 1765 12,461,676

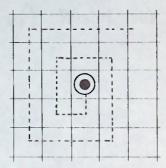
Raab's calculations were done in Applesoft on a 16K Apple II, in less than 40 minutes.



A Pattern Of New Distances

In the grid of squares shown on the facing page, a pattern is formed by the following scheme:

The center square (shown circled) is selected. The other squares in the grid are then sampled in spiral fashion:



For each square sampled, the distance to all previously selected squares is calculated. If <u>any</u> of these distances is new, then the square is selected, marked as shown, and entered into a table of selected squares. For those squares for which <u>all</u> the distances to <u>all</u> previously selected squares are already in the list of distances, the square is by-passed.

An endless series of such patterns can be created (and creating them is an excellent coding exercise in almost any computer language), by varying:

- The scanning scheme. For the pattern shown, a clockwise spiral scan was used. One can imagine diagonal scans, TV-type scans, and so on.
- The pattern can be "seeded" with additional fixed selected squares.

For computers whose normal output is to a screen, the production of such patterns can be plotted as the points are selected. For each pattern, the first few points are found rapidly, but the time for each successive point gets longer.

PROBLEM 282

Inventory Clearance Sale

Limited Time Only

Complete Your File of Popular Computing

On orders received up to January 31, 1981:

- 1. For 10 or more of the issues still available.
- 2. 50¢ (U.S.) per copy.
- 3. Chosen from issues 1 through 82 only.
- 4. Mailed to one address by surface mail.

MAR MAY JUL JUL SEP OCT OCT DEC 1 2 3 4 5 **X** X 9 Vol. 1 1973 14 X X X 19 X 21 10 11 12 Vol. 2 1974 27 28 💥 30 23 24 25 26 31 32 33 Vol. 3 1975 **X X** 36 37 38 39 40 41 Vol. 4 1976 42 43 44 45 47 48 Vol. 5 1977 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 Vol. 6 1978 70 71 72 73 74 75 76 77 78 79 80 81 Vol. 7 1979 82 83 84 85 86 87 88 89 Vol. 8 1980 90 91 92 93